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MATHEMATICAL MODEL OF GAS BLOWING THROUGH MOLTEN GLASS VIA A TUYERE LOCATED BELOW THE MELT LEVEL

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A mathematical model of the process of blowing high-temperature products of fuel combustion through molten glass in the melting tank of a glassmaking furnace via side tuyeres placed below melt level is presented. Blowing is conducted in two regimes — bubble and jet.

Key words: molten glass, blowing through, modeling, mathematical model, hydrodynamics, bubbling layer, gas bubble.

Gas can be blown through molten glass via a tuyeres, located below the melt level, in two regimes — bubble and jet. In the first case the gas at the tip of a tuyere forms a gas cavity which is held on the tip by surface tension. The expulsive buoyancy force strives to detach the cavity. As the volume of the gas cavity increases the buoyancy force increases, while the surface tension remains unchanged. In the limiting case these forces are equal to one another. As it continues to grow the cavity detaches from the tip of the tuyere, and in the process a gas bubble forms and floats upward in the melt tank.

In the second case, where a gas jet flows into the molten glass, the gas at the exit from the tuyere forms a stable gas cavity, the flow in which forms a gas jet. Individual bubbles detach from the surface of the gas jet, while the cavity itself is continually sits on the tip of the tuyere.

In the present work we examine the case where high-temperature products of fuel combustion are blown into the molten glass in the melt tank through a side tuyere placed below the melt level.

The following assumptions are made to construct a model of the blowing process:

– the gas and liquid are chemically neutral, isothermal and incompressible;

– the molten glass is a homogeneous viscous liquid;

– before detaching from the tip of the tuyere a gas bubble is symmetric relative the longitudinal axis of the tuyere;

– the gas jet is symmetric relative to the longitudinal axis of the tuyere over the entire distance to breakup into bubbles;

– the minimum static gas pressure at the exit from tuyere equals the static pressure in the furnace space above the molten glass.

The assumption of the chemical neutrality of the gas and liquid is adopted because in a physical model it is impossible to simulate simultaneously the hydrodynamics and mass transfer accompanied by a chemical reaction.

The gas and liquid are assumed to be incompressible because the pressure in the bubbling layer varies within negligible limits while the velocity of these phases is relatively low.

For a number of technological apparatuses with a bubbling layer the temperatures of the gases flowing into and out of the layer differ by not more than $10 – 15\%$, i.e., the gas and liquid in the layer are practically isothermal. In cases where a cold gas is blown into a tank of molten glass the temperature of the gas becomes equal to the temperature of the molten glass within less than two tuyere lengths. In other words, even in this case the gas and liquid in most of the interior volume of the layer interact as isothermal media. On the initial section, where the gas temperature increases, the velocity of the gas increases, which should intensify mixing of the liquid in a zone near the tuyere. In this connection a study of the efficiency of heat and mass transfer using an isothermal hydraulic model corresponds to the least favorable conditions for the process to proceed.

It is known that the mass content of the disperse phase (solid particles or drops of insoluble liquid) in a bubbling layer does not exceed 10%[3] [1]. Impurity content to 20% of the total mass of the melt has no appreciable effect on the hydrodynamics of the bubbling layer [2]. Therefore, it can be assumed that the viscous liquid (molten glass) is homogeneous in the melt tank.

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[3] Here and below the content by weight.

When gas is blown through a side tuyere in the bubble regime a growing gas bubble moves upward relative to the longitudinal axis of the tuyere. At the moment of detachment the bubble is located above the top edge of the tuyere. As a result the gas pressure inside the bubble drops by the amount

$$\Delta p = g\rho_{gas} d_0 / 3,$$

where g is the acceleration of free fall, m/sec^2; ρ_{gas} is the density of the gas, g/m^3; and, d_0 is the diameter of the opening of the tuyere, m.

The gas pressure inside the bubble is given by

$$p_g = g\rho_{liq} h_{liq},$$

where ρ_{liq} is the density of the liquid (molten glass), kg/m^3, and h_{liq} is the height of the layer of liquid above the axis of the tuyere, m.

Since p_g is at least three orders of magnitude greater than Δp, any change of the gas pressure inside a bubble can be neglected and it can be assumed that as it grows a bubble remains symmetric relative to the longitudinal axis of the tuyere.

The sum of the dynamic and static gas pressures at the exit from the tuyere is a constant, equal to the sum of the counter pressure of the molten glass and turbulent pulsations. In accordance with [1], the energy of the turbulent pulsations is practically a constant; during blowing the ratio between the dynamic and state pressures varies correspondingly. At the moment the gas cavity detaches from the tip of tuyere the liquid covers the entrance opening of the tuyere, and the gas stops for an instant. At this moment the static pressure of the gas has its maximum value equal to the stopping pressure:

$$(p_{st}^{gas})^{max} = p_0 + g\rho_{liq} h_{liq},$$

where p_0 is the static pressure on the melt surface, Pa, and h_{liq} is the height of the fined (gas-free) liquid above the axis of the tuyere, m.

At the next instant a gas cavity starts to form, under the pressure of the gas, in front of the tuyere tip. The gas begins to move, and as the cavity grows in size its velocity increases. The total pressure developed by the gas at the tuyere exit is

$$p_{st}^{gas} + \frac{\rho_{gas} w_{gas}^2}{2} = p_0 + g\rho_{liq} h_{liq}.$$

Before the next bubble detaches from the tip of the tuyere the gas velocity w_{gas} assumes its maximum value and the static gas pressure its minimum value.

The assumption that the minimum static gas pressure at the tuyere exit equals the static gas pressure in the furnace space above the molten glass $(p_{st}^{gas})^{min} \approx p_0$ holds if

$$\frac{\rho_{gas} w_{gas}^2}{2} \geq \rho_{liq} g h_{liq},$$

i.e., when

$$w_{gas} \geq \sqrt{\frac{\rho_{liq}}{\rho_{gas}} 2g h_{liq}}.$$

Taking $\rho_{liq} = 2300$ kg/m^3, $h_{liq} = 1.0$ m and $\rho_{gas} = 1.29$ kg/m^3 we find that this assumption definitely holds for $(w_{gas})^{max} \geq 189$ m/sec. Since the velocity obtained is close to the average outflow velocity of the gas in most furnaces with a bubbling layer, this assumption is correct.

We now represent the mathematical model of the hydrodynamics of the bubbling layer in the following form:

$$
\left.
\begin{aligned}
&\frac{\partial \vec{w}_{gas}}{\partial t} = \vec{k} - \frac{1}{\rho_{gas}} \operatorname{grad} p_{gas} + \nu_{gas} \nabla^2 \vec{w}_{gas}; \quad &(1)\\
&\operatorname{div} \vec{w}_{gas} = 0; \quad &(2)\\
&\frac{\partial \vec{w}_{liq}}{\partial t} = \vec{k} - \frac{1}{\rho_{liq}} \operatorname{grad} p_{liq} + \nu_{liq} \nabla^2 \vec{w}_{liq}; \quad &(3)\\
&\operatorname{div} \vec{w}_{gas} = 0; \quad &(4)\\
&\rho \frac{\partial \vec{w}_{liq\text{-}gas}}{\partial t} = \vec{g}(\rho_{liq} - \rho_{gas}) - \operatorname{grad} p_{liq}\\
&\quad + \mu_{gas} \nabla^2 \vec{w}_{liq\text{-}gas}; \quad &(5)\\
&\operatorname{div} \vec{w}_{liq\text{-}gas} = 0; \quad &(6)\\
&p_{liq}^{if} - 2\mu_{liq}\left(\frac{\partial \vec{w}}{\partial Y}\right)_{liq\text{-}gas}^{if} = p_{gas}^{if} + \sigma\left(\frac{1}{R_1} + \frac{1}{R_2}\right); \quad &(7)\\
&\vec{w}_{gas}^{if} = \vec{w}_{liq}^{if} = \vec{w}_b, \quad &(8)
\end{aligned}
\right\} \text{(I)}
$$

where $\vec{w}_{gas}$, $\vec{w}_{liq}$ are, respectively, the velocity of the gas and liquid, m/sec; $\vec{k}$ is the mass density of the external mass force (gravity), N/kg; ρ_{gas}, ρ_{liq} are the density of the gas and liquid, respectively, kg/m^3; p_{liq} is the pressure of the liquid at the surface of a bubble, Pa; p_{gas} is the gas pressure inside the bubble, Pa; $\operatorname{grad} p$ is the gradient of the pressure, Pa/m; ν_{gas}, ν_{liq} are, respectively, the kinematic coefficients of viscosity of the gas and liquid, m^2/sec; $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ is the Laplace operator; $\operatorname{div}(\vec{w}) = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$ is the divergence of the velocity vector; $\vec{w}_{liq\text{-}gas}$ is the relative velocity of laving of the gas bubble by the molten glass, m/sec; $\vec{g}$ is the acceleration of free fall, m/sec^2; p_{gas}^{if}, p_{liq}^{if} are the pressure of the gas and liquid on the phase interface, Pa; μ_{gas}, μ_{liq} are the dynamic coefficient of viscosity of the gas and liquid, respectively, Pa · sec; σ is the interphase tension at the gas bubble – liquid interface, N/m; R_1, R_2 are the principal radii of curvature of the surface of the rising bubble (the bubble is assumed to be an ellipsoid), m; and, w_b is the rise velocity of the bubble, m/sec.

The uniqueness conditions are formulated separately for the bubble and jet regimes of outflow:

for the bubble regime:

$$
\left.\begin{array}{lll}
- \text{ for } t \ge t_0 & 0.5\rho_{gas}\left(\dfrac{\partial x}{\partial t}\right)^2 = \vec{g}\rho_{liq}h_{liq}; \\
- \text{ for } X = 0 & Z = 0.5d_b, & w_{z\,gas} = 0; \\
- \text{ for } Z = 0 & X = 0.5d_b, & w_{x\,gas} = 0;
\end{array}\right\} \quad \text{(II)}
$$

for the jet regime:

$$
\left.\begin{array}{ll}
- \text{ for } t = t_0 & \vec{w}_0 = \vec{w}_{cr}; \\
- \text{ for } 0 < X < l_{jet} & Z = 0.5d_{jet}; \\
\multicolumn{2}{l}{(\rho_{gas} + \rho_{liq})\dfrac{\partial^2\xi}{\partial t^2} + \dfrac{1}{\lambda^2}(\mu_{gas} + \mu_{liq})\dfrac{\partial\xi}{\partial t} +} \\
\multicolumn{2}{l}{\dfrac{1}{\lambda}[\vec{F} + (\rho_{gas} - \rho_{liq})\vec{b}\cos(\omega t)]\xi = 0;} \\
\multicolumn{2}{l}{\vec{F} = \vec{g}(\rho_{gas} - \rho_{liq}) + \dfrac{\sigma}{\lambda^2};} \\
- \text{ for } X = l_{jet} & 0.5\rho_{gas}\left(\dfrac{\partial x}{\partial t}\right)^2 = \vec{g}\rho_{liq}h_{liq},
\end{array}\right\} \quad \text{(III)}
$$

where t_0 is the moment of onset of bubble growth or a transition of the bubble into the jet outflow regime; d_b is the bubble diameter, m; d_{jet} is the diameter of the transverse section of the jet, m; $\vec{w}_0$ is the average flow velocity of the gas in the exit section of the tuyere, m/sec; $\vec{w}_{cr}$ is the velocity at which the jet outflow regime commences, m/sec; l_{jet} is the length of the jet in the melt, m; ξ is the displacement along the Z axis of the fluctuating gas jet – molten glass interface; λ is the wavelength of these oscillations, m; $\vec{b}$ is the amplitude value of the fluctuating acceleration, acting along the vertical axis Z and due to the oscillations of the jet – molten glass interface, m/sec²; and, ω is the circular frequency of the oscillations, rad/sec.

We shall assume the outflow to be quasistationary. Then, under the uniqueness conditions the following are also prescribed:

– the geometric parameters of the furnace (interaxial distance of the tuyeres l_{tu}, m; diameter of the outflow opening of a tuyere d_0, m; height of the layer of fined molten glass above the axis of the tuyere h_{liq}, m);

– the physical parameters of the molten glass and gas (ρ, kg/m³; μ, Pa · sec; σ, N/m);

– the gas velocity at the exit from a tuyere w_0, m/sec;

– the sound velocity in the gas a, m/sec.

The equations (1) – (4) of the system (I) are the equations of motion and continuity of the gas and liquid. The equations (5), (6) are the equations of motion and continuity of the liquid flowing around a gas bubble, conventionally assumed to be stationary. The equation (7) expresses the condition that the normal stresses on the surface of a bubble taking account of the surface tension and neglecting the viscosity of the gas are equal. The equation (8) expresses the absence of phase slip at the surface of a bubble.

The physical meaning of the system (II) is as follows. In the bubble regime of outflow, at the moment a bubble detaches liquid momentarily covers the opening. in the exit section of a tuyere, the gas moving along the delivery channel is suddenly decelerated right up to complete stoppage, and its kinetic energy is converted into a static head. This assumes that $h_{liq} \gg d_0$. The symbol X denotes the distance along the longitudinal axis of the tuyere at which gas decelerates from the velocity w_0 to zero and t is the time during which this deceleration occurs.

The system (III) describes the mechanism of the suppression of the expulsive force on the surface of the gas jet due to the turbulent pulsations of the gas.

The condition that the dynamic pressure of the gas flow equals the static pressure of the molten glass in the melt tank of the furnace always holds in the jet outflow regime at the end of the jet. It is expressed by the last equation of the system (III).

The mathematical model presented above is the theoretical basis for developing the methods and apparatus for physical modeling of mixing processes in molten glass through which a gas jet, located on the side wall of the furnace below the melt level, is blown.

REFERENCES

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